CHAPTER 1

INTRODUCTION

"If God had wanted us to use the metric system, he would
have given us ten fingers and ten toes"
Anon.

1.1 BACKGROUND

The measurement of length and the provision of length standards are of fundamental importance to any technologically developed society. The ability to measure length to a required accuracy and demonstrate that the measurement has been performed in terms of universally recognised units underlies much of the world's trade. The world has seen many standards of length in use throughout history [**1**] ranging from the simple use of thumbs and feet, to the more advanced definitions relating to the wavelength and speed of light. One common theme is the continuing refinement of standards, leading to more specific definitions and more accurate methods of realising them.

Perhaps the most significant step forward in the field of length standards was made by Michelson at the end of the nineteenth century, when he measured the wavelength of the red light from cadmium in terms of the International Prototype Metre bar [**2**]. This established the techniques of interferometric comparison whereby material lengths are measured in terms of known wavelengths of light. The work of Michelson and his contemporaries stimulated international development of wavelength standards which resulted in the adoption in 1960 of a new definition of the metre, based on the wavelength of light from a krypton lamp [**3**]. The development of the laser in the 1960s produced a new wavelength source which, with careful control, could surpass the stability and accuracy of the krypton wavelength standard. This prompted a new definition of the metre [**4**] in 1983 which, although it abandoned the direct concept of wavelength in the definition, recommended the use of laser wavelengths in the realisation of the metre.

At the time of writing, this definition still stands and the realisation of the metre at the National Physical Laboratory (NPL) as a wavelength standard using iodine-stabilised helium-neon lasers serves as the UK's primary length standard.

1.2 THE MEASUREMENT OF LENGTH

1.2.1 What is length?

Whilst it is quite difficult to say exactly what is meant by ‘length’, it is relatively easy to actually measure length, especially the length of material objects. Measurements of length range from the dimensions of atoms and their constituents to the size of the visible universe. Units used to describe this wide range of lengths include parsecs, light years, solar diameters, light seconds, kilometres, metres, yards, feet, inches, millimetres, microns, thousandths of an inch, micrometres, nanometres, angstroms and picometres. Each unit or sub-multiple of a unit must be connected to each other unit if the measurements at different scales are to be related. This necessitates a standardised system of units, with consistent definitions and interrelationships. The most common such system in use today is the Système International d’Unités (SI).

1.2.2 The International System of units (SI)

In the second half of the nineteenth century the inch, yard and foot were the most common units in use in Britain, but the centimetre, gram and second were also in use. These units, called the “CGS electromagnetic system” were coherent, *i.e.* there were no numerical factors other than unity used in the definitions of the derived units. There was another set of units, the “CGS electrostatic system” which was used for measurements of charge, potential and capacitance. The problem with this latter system of units was that the sizes of the units were inconvenient. In 1881 an international agreement defined new units: the volt as 10^8 CGS potential units, the ohm as 10^9 CGS resistance units and the ampere as 0.1 CGS units.

These new units were mutually coherent but were not coherent with the magnetic or mechanical units. This led Giorgi in 1902 to propose a new set of units based on the metre, kilogram, second and ampere. This allowed the magnetic field strength to be expressed in amperes per metre thus removing a factor of π from most electromagnetic formulae involving rectilinear geometry, and transferring it to formulae using cylindrical or spherical geometry.

In 1948 the 9th Conférence Générales des Poids et Mesures (CGPM) adopted these mechanical units. The 11th CGPM later added to these the candela and kelvin and the supplementary units the radian and steradian. In 1971 the 14th CGPM added the mole as the amount of substance, completing the set of 7 base units.

Quantity	Unit	Symbol
Time	second	s
Length	metre	m
Mass	kilogram	kg
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	Cd
Amount of substance	mole	mol

***Table 1.1** - The 7 base units of the SI system*

1.2.3 The definitions of the SI units

The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.

The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.

The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.

The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.

The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of (1/683) watt per steradian.

The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12.

The *supplementary* units are defined thus:

The radian is the plane angle between two radii of a circle which cut off on the circumference an arc equal in length to the radius.

The steradian is the solid angle which, having its vertex in the centre of a sphere, cuts off an area of the surface of the sphere equal to that of a square with sides of length equal to the radius of the sphere.

From these basic units, one can derive other units to describe any quantity which can be measured, *e.g.* velocity as the rate of change of length in unit time (m s^{-1}). For a brief history of the units, see for example Kaye & Laby [**5**] or The International System of Units [**6**].

Some of these base units may be considered less fundamental than the others as they contain references to other units in their definitions. Thus the metre relies on the definition of the second, the ampere on the metre. The definition of the kilogram is also somewhat unusual in that it is the only unit which is derived from a physical object. It is also unfortunate that the unit is the *kilo*gram, rather than the gram, and thus contains one of the recommended prefixes used to denote multiples and sub-multiples of units.

Unit	Realisation	Accuracy of realisation
second	Caesium beam clock	1 in10^{13}
metre	Wavelength of laser	2.5 in 10^{11}
kilogram	British copy no. 18	1 in 10^{9}
ampere	Via the Watt	8 in 10^{6}
kelvin	Water triple point cells	1 in 10^{4}
mole	Directly from definition	(Avogadro const: 6 in 10^{7})
candela	Cryogenic radiometer	1 in 10^{3}

Table 1.2 *- Realisations of the SI units at NPL*

For most measurements, only 5 of these base units are required: length, time, mass, amount of substance and electrical current. If required, temperature can be defined in terms of energy and hence in terms of mass, length and time, without requiring its own separate unit. The unit for radiated intensity can be expressed similarly as watts per square metre per steradian.

1.3 HISTORICAL UNITS OF LENGTH

1.3.1 Timetable of events

3000 BC Egyptian & Mesopotamian cubit in common use

12 BC Lower Germany recognises 'northern cubit' (just over 2 feet in length)

410 Anglo-Saxon foot used throughout Britain

1305 Edward I decreed that "3 dry round grains of barley makes 1 in, 12 inches equals 1 foot, 3 feet equal 1 ulna"

1497 Size of ulna (yard) of Henry VII same as modern to within 0.04 inch

1588 Elizabeth I yard same as modern yard to within 0.01 inch

1742 Two brass bars constructed, 42 in x 0.5 in x 0.25 in - yard engraved on surface

1758 Royal Commission start work on new bronze standard, 1 in square, gold inserts

1760 New bronze yard standard completed

1790 Talleyrand (Bishop of Autun) proposes new system of lengths for France

1791 Academy of France Commission set up to consider a new decimal scale of units

1791 Commission chose quadrant of meridian of Earth, terrestrial pole to equator, as basis for length standard, rejecting use of a pendulum beating the second

1792 Delambre and Méchain commence measurements along meridian

1793 National Convention (France) adopts 1 m = 443.44 lignes of Toise de Pérou

1795 Basic law passed adopting metric system passed by Convention

1799 Mètre des Archives, (platinum), 25.3 x 4 mm constructed and adjusted by Janetti to be 1/10 000 000 of Earth quadrant,

1824 Royal Commission bar (yard) adopted as new primary standard for UK

1829 Babinet suggests use of wavelengths as length standards

1834 Fire in Houses of Parliament destroys yard standard

1837 France rescinded all weights and measures other than the metric system

1843 Sheepshanks & Bailey of British Royal Commission work on new bronze yard standard

1855 Work completed on 1 inch square bronze yard standard replacement

1864 Metric system sanctioned in UK by Act of Parliament

1872 30 new prototype metres construction started (X cross-section) 90% Pt, 10% Ir, based on design by Tresca

1875 Convention du Mètre signed

1878 Weights and Measures Act - metric system fully legalised in UK

1884 Britain signs Convention du Mètre

1889 Work on 30 prototype metres completed, deposited at BIPM

1889 Definition of metre in terms of Prototype Metre

1893 Michelson & Benoit compare Cd red line to Prototype Metre

1893 American yard linked to metre

1899 Metre Copy no. 16 measured as 0.999 999 400 m

1899 Bill and Order in Parliament, legalising metre copy no. 16 as UK metre

1922 Comparison of metre and yard (inch) : 1 m = 39.370 147 inches

1927 Re-definition of the metre

1956 Engraved lines on copy no. 16 re-ruled to allow measurements at 0 °C and 20 °C

1959 New value for inch : 1 inch = 25.4 mm exactly, 1 m = 39.370 079 inches

1960 Re-definition of metre in terms of krypton-86 wavelength (± 4 x 10^{-9})

1963 Weights and Measures Act legalises new definition of inch as 2.54 cm

1975 Speed of light fixed at 299 792 458 m s^{-1}

1983 Definition of metre as distance travelled by light in 1/299 792 458 s (absolute)

1983 Accuracy of second ± 1 x 10^{-13}, accuracy of metre realisation as wavelength :
± 2 x 10^{10} to ± 2 x 10^{9} for lasers stabilised to saturated absorption, 10^{8} for others

1.3.2 The first definition of the metre

The first Conférence Générales des Poids et Mesures [**4**] (CGPM) in 1889 stated of the Prototype Metre, that:

<<Ce prototype représentera désormais à la temperature de la glace fondante, l'unité métrique de longeur.>>

"This prototype, at the temperature of melting ice, will henceforth represent the unit of length."

This was a rather informal definition as it did not include certain details concerning how the bar was to be supported.

(Note that the official language of the SI system of units is French: the above wording in English is only an approved translation, NOT the definition. The language difference can sometimes be subtle, but important. For instance in a later definition, use is made of the word "vide" or "vacuum". In English, the phrase "free space" is more often used to indicate the absence of cosmological matter such as black holes and virtual particles, however there is no distinction in French between "vacuum" and "free space" [**7**]).

1.3.3 The 1927 definition of the metre

In 1927 the 7th CGPM adopted a new definition [**3**] of the metre:

<<L’unité de longeur est le mètre, defini par la distance, à 0°, des axes de deux traits médians tracés sur la barre de platine iridié déposée au Bureau International des Poids et Mesures et déclarée Prototype du mètre par la première Conférence Générale des Poids et Mesures, cette règle étant soumise à la pression atmosphérique normale et supporté par deux rouleaux d’au moins un centimètre de diamètre, situés symétriquement dans un même plan horizontal et à la distance de 571 mm l’un de l’autre.>>

“The unit of length is the metre, defined as the distance at 0° between the two lines engraved in the platinum iridium bar, deposited in the BIPM, and declared as the Prototype metre by the 1st CGPM, this standard supported at normal atmospheric pressure on two rollers, of less than 1 cm diameter, situated symmetrically in a horizontal plane, at a distance of 571 mm from each other.”

Thus the first formal definition of the metre was based on a single material object (as the kilogram is today). This bar had an ‘X’-shaped cross section designed by Tresca [**3**] to have maximum rigidity with minimum use of material, with the 2 lines defining the metre engraved on the neutral plane or surface of the bar, *i.e.* the surface which experiences no net compression or expansion when the bar is supported on its designated rollers. The positions of the rollers corresponded to the “Bessel points” of the bar, positioned 571 mm apart, either side of the centre. When supported at these points, the length of the bar is unchanged from its free state.

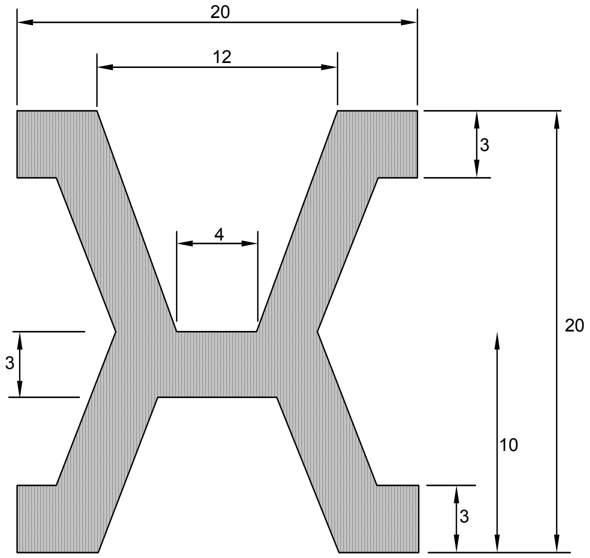

Figure 1.1 *- Cross-section of International Prototype Metre (and copies), based on a design by Tresca, manufactured between 1882 and 1889 (dimensions in mm)*

Although it was discovered that the original Mètre des Archives (an End Standard) was short by 0.23 mm of the size it was meant to be, namely one ten-millionth of the Earth's quadrant (realised on a meridian connecting Dunkirk and Barcelona), the new Prototype Metre was made to be the same length as its predecessor to avoid any change to the physical size of the metre which was in use.

1.3.4 The 1960 definition of the metre

In 1893 Michelson and Benoit working at the Bureau International des Poids et Mesures used an interferometer to measure the wavelength of the cadmium red line in terms of the metre. Following the definition of the metre in 1927 progress was made in the determination of the wavelengths of emission lines and also in the purification of isotopes of elements, notably krypton 86, cadmium 114, and mercury 198. This allowed the production of wavelengths from emission lines with suitably narrow spectral lines, the stability and reproducibility of which could surpass the values obtainable from the Prototype Metre. Thus in 1960 the CGPM adopted a new definition of the metre [**3**] based on wavelengths:

<<

1. *Le mètre est la longeur égale à 1 650 763.73 longueurs d'onde dans le vide de la radiation correspondant à la transition entre les niveaux* $2p_{10}$ *et* $5d_5$ *de l'atome de krypton-86.*
2. *La Définition du Mètre en vigeur depuis 1889, fondée sur le Prototype International en platine iridié, est abrogée.*
3. *Le Prototype International du Mètre sanctionné par la Première Conférence Générale des Poids et Mesures de 1889 sera conservé au Bureau International des Poids et Mesures dans les mêmes conditions que celles qui ont été fixées en 1889.>>*

which translates as

"

1. The metre is the length equal to 1 650 763.73 wavelengths in vacuum of the radiation corresponding to the transition between the energy levels $2p_{10}$ and $5d_5$ of an atom of krypton 86.
2. The definition of the metre in use since 1889, based on the International Prototype of platinum iridium, is abrogated.
3. The International Prototype Metre sanctioned by the first CGPM of 1889 will be conserved at the BIPM in the same conditions that it was placed in 1889."

The source of the standard radiation recommended by the 1960 Committee was the Engelhard lamp, operated according to the following instructions [**3**]:

<< Conformément au paragraphe 1 de la Résolution 2 adoptée par la Onzième Conférence Générales des Poids et Mesures (octobre 1960), le Comité International des Poids et Mesures recommande que la radiation du krypton 86 adoptée comme étalon fondamental de longueur soit réalisée au moyen d'une lampe à décharge à cathode chaude contenant du krypton 86 d'une pureté non inférieure à 99 pour cent, en quantité suffisante pour assurer la présence de krypton solide à la température de 64 °K, cette lampe étant munie d'un capillaire ayant les caractéristiques suivantes: diamétre intérieur 2 à 4 millimètres, épaisseur des parois 1 millimètre environ.

On estime que la longueur d'onde de la radiation émise par la colonne positive est égale, à 1 cent-millionième (10^{-8}) près, à la longueur d'onde correspondant à la transition entre les niveaux non perturbés, lorsque les conditions suivantes sont satisfaites:

1. *le capillaire est observé en bout de façon que les rayons lumineux utilisés cheminent du côte cathodique vers le côte anodique;*
2. *la partie inférieure de la lampe, y compris le capillaire, est immergée dans un bain réfrigérant maintenu à la température du point triple de l'azote, à 1 degré près;*
3. *la densité du courant dans le capillaire est 0,3 ± 0.1 ampère par centimètre carré. >>*

"Conforming to paragraph 1 of resolution 2 of the 11th CGPM (October 1960), the CIPM recommends that the radiation of krypton 86 adopted as the fundamental standard of length should be realised by means of a hot cathode discharge tube containing krypton 86 of a purity not less than 99 per cent in a sufficient quantity to ensure the presence of solid krypton at a temperature of 64 K; the lamp having a capillary with the following characteristics: internal diameter 2 to 4 mm, wall thickness about 1 mm.

It is believed that the wavelength of the radiation emitted by the positive volume is equal to within about one hundred millionth (10^{-8}) of the wavelength corresponding to the transition between the unperturbed levels, provided the following conditions are satisfied:

1. the capillary is observed end-on in such a way that the rays used travel from its cathodic to its anodic end
2. the lower part of the lamp, including the capillary, is immersed in a cooling bath maintained at the temperature of the triple point of nitrogen within about 1 degree
3. the current density in the capillary is 0.3 ± 0.1 amperes per square centimetre."

The number of wavelengths in the definition was chosen to correspond as closely as possible with the previous definition. By removing the dependence on a single material object, the new definition allowed greater access for metrologists to a more accurate length standard. However the variation in the wavelength due to impurities in the krypton and other constructional and operational details required extra "instructions for use" as recommended by the CIPM.

1.3.5 The present (1983) definition of the metre

The first simultaneous measurements of both the frequency and wavelength of light were performed in 1972 leading to a more accurate value for the speed of light [**8**] (see figure 1.2).

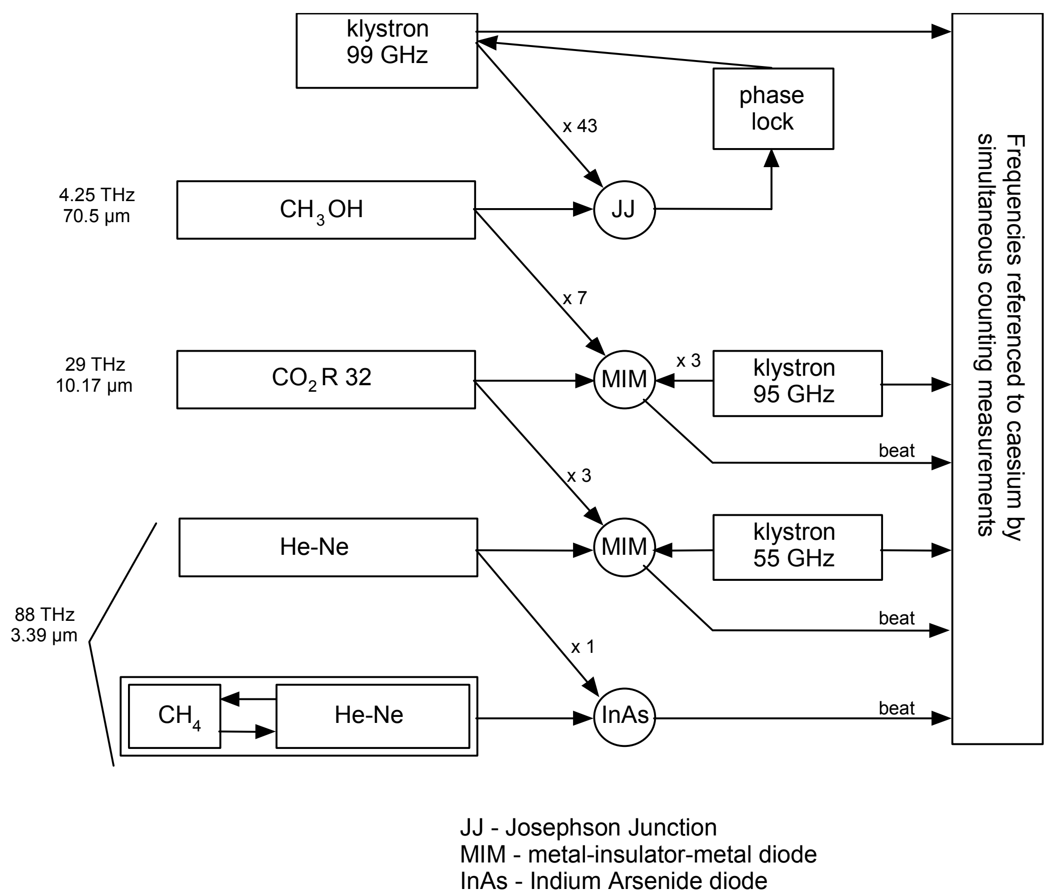

Figure 1.2 *- Frequency chain for methane laser frequency determination*

The uncertainty in this value was estimated to be 1.2 m s^{-1}. This value was used to link the frequencies and wavelengths of other lasers. The weak link in the chain was found to be the realisation of the metre using the krypton 86 lamp, which had an uncertainty similar to that of the measurement of the speed of light. Also, by the early 1970s the frequencies of lasers could be compared with greater accuracy than the results could be expressed in absolute units. It seemed that a more accurate definition of the speed of light was required. To prevent discrepancies between different experiments, the Comité Consultatif pour la Définition du Mètre (CCDM) recommended a value for the speed of light which was fixed in 1975 by the 15th CGPM at 299 792 458 m s^{-1}. This implicitly contained a new definition of the metre which at the level of uncertainty within 4 parts in 10^9 could cause discrepancies with previously determined wavelengths, with the danger that a laser wavelength scale might have become established, separate from the SI system.

To avoid these problems, in 1983 the CCDM proposed a new definition for the metre based on the speed of light. This was then adopted by the Comité Consultatif des Unités (CCU), the Comité International des Poids et Mesures (CIPM) and the 17th Conférence Générale des Poids et Mesures (CGPM) in October 1983. The definition was chosen to be both intelligible enough to be understood by physics students and yet be precise enough to allow metrologists working at the frontiers of measurement to use it as a working definition. Thus the definition was kept as simple as possible, with an additional recommendation of how to use it in practice [**4**]:

<<Le mètre est la longeur du trajet parcouru dans le vide par la lumière pendant une durée de 1/299,792,458 de seconde.>>

<<la définition du mètre en vigeur depuis 1960, fondée sur la transition entre les niveaux 2p$_{10}$ et 5d5 de l'atome de krypton 86, soit abrogée.>>

"The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second."

"The definition of the metre in use since 1960, based on the transition between the two lines 2p$_{10}$ and 5d$_5$ of the krypton atom, is abrogated."

RECOMMENDATION 1(CI-1983)

The Comité International des Poids et Mesures
recommends
- that the metre be realised by one of the following methods:

(a) by means of the length *l* of the path travelled in vacuum by a plane electromagnetic wave in a time *t*; this length is obtained from the measured time

t, using the relation $l = c.t$ and the value of the speed of light in a vacuum $c = 299\,792\,458$ m s^{-1};

(b) by means of the wavelength in vacuum λ of a plane electromagnetic wave of frequency f; this wavelength is obtained from the measured frequency f, using the relation $\lambda = c/f$ and the value of the speed of light in vacuum $c = 299\,792\,458$ m s^{-1};

(c) by means of one the radiations from the list below, whose stated wavelength in vacuum, or whose stated frequency, can be used, provided that the given specifications and good practice are followed;

Laser	Absorber	Trans^n, line, comp^t	f/MHz	λ/nm
He-Ne	CH_4	n_3,P(7),F_2	88 376 181.608	3392.231 397
He-Ne	$^{127}I_2$	17-1,P(62),o	520 206 808.51	576.294 760 27
He-Ne	$^{127}I_2$	11-5,R(127),i	473 612 214.8	632.991 398 1
He-Ne	$^{127}I_2$	9-2,R(47),o	489 880 355.1	611.970 769 8
Ar^+	$^{127}I_2$	43-0,P(13)a_3	582 490 603.6	514.673 466 2

Table 1.3 *- Recommended wavelengths for realisation of the metre*

1.3.6 Limitations of the present realisation of the metre

The first limitation of the current realisation of the metre lies in its dependence on the speed of light. The value 299 792 458 m s^{-1} is the latest and most accurate result with an uncertainty of ± 1.2 m s^{-1} and is based on measurements of the frequency and wavelength of a helium-neon laser radiation, stabilised to an infra-red transition in methane at about 3.39 µm, with later confirmation at a wavelength of 9.3 µm using a stabilised carbon dioxide laser [**9**]. Although the definition is fixed, the realisation in absolute terms contains an uncertainty at the level of 4 parts in 10^9 due to the uncertainty in the speed of light. This has been recognised by the CGPM whose recommendation is that any changes in the measured value of the speed of light will be ascribed to discrepancies between the maintained metre and the metre of the SI definition.

The second source of uncertainty lies in the uncertainty in the realisation of the second through the use of the caesium beam clock, which is 1 in 10^{13}. This directly affects the uncertainty in the frequency of the iodine-stabilised laser, which in turn affects the wavelength.

It is assumed that during any realisation of the metre, the experimenter will seek to take account of any effects of relativity or other influences which would affect either the duration of the second or the speed of light.

1.3.7 Future realisations of the metre

The first definition of the metre in terms of the metre bar stood for 130 years until it was replaced with the second definition in terms of wavelengths. This in turn was replaced after only 23 years with the current definition in terms of light path. How long will the current definition stand? Looking at the increasing accuracy of length measurements and the corresponding reduction in the uncertainty of the realisation of the metre [**7**], as shown in figure 1.3, it appears that a new realisation will probably be required early in the next century. In particular, the demands of nanometrology [**10**,**11**] and the increasing accuracy of commercial interferometers which are now accurate to between 10^{-8} and 10^{-9} make a more accurate realisation a necessity in 10 to 20 years time.

According to Petley [**12**], the whole SI system is dynamic. During the last two decades the accuracy with which the fundamental constants can be measured has caught up with the accuracy to which the definitions of many of the base units can be realised. In many cases fundamental constants are now either part of the definition of a base unit or used to maintain a reproducible secondary unit, *e.g.* the Josephson effect as a voltage standard. In principle it should thus be possible for users to realise their own base units in terms of these fundamental constants, however this is not yet the case: commercial atomic clocks and helium-neon lasers still require calibration against primary standards.

The ability to measure physical quantities has improved over the last decade, sometimes at the rate of a tenfold increase in accuracy per decade. This progress has actually been made in discrete steps. There is a time lag of approximately 10 years between the advancement of a new accuracy of calibration and the routine achievement of the same accuracy in commercial products. Similarly, work in national standards laboratories is at an accuracy above that of the commercial sector, but below the most accurate possible. As the accuracy of commercial instruments approaches that of calibrations offered by national standards laboratories and the accuracy of those calibrations approaches that of the national standard, the standard is replaced by a newer one of greater accuracy, and the cycle starts again.

Note also that it is a new realisation of the metre that will be required, not a new definition. The current definition is sufficiently open-ended, being based on a fundamental constant, to allow future methods of realising the metre to be incorporated, without the need to change the definition. In particular new sources will be added to the list of recommended radiations and their uncertainties will be reduced as new frequency measurements are made. Thus the realisation of the unit of length will be kept up to date, delaying the time when a new definition will be needed. This does however lead to an extensive list of recommended wavelengths and some rationalisation may be required as older sources become less commonly used.

Note also that the definition does not mean that the speed of light can never change; rather that, if it does, then the size of the metre will change in accordance so that the numerical value 299 792 458 is preserved.

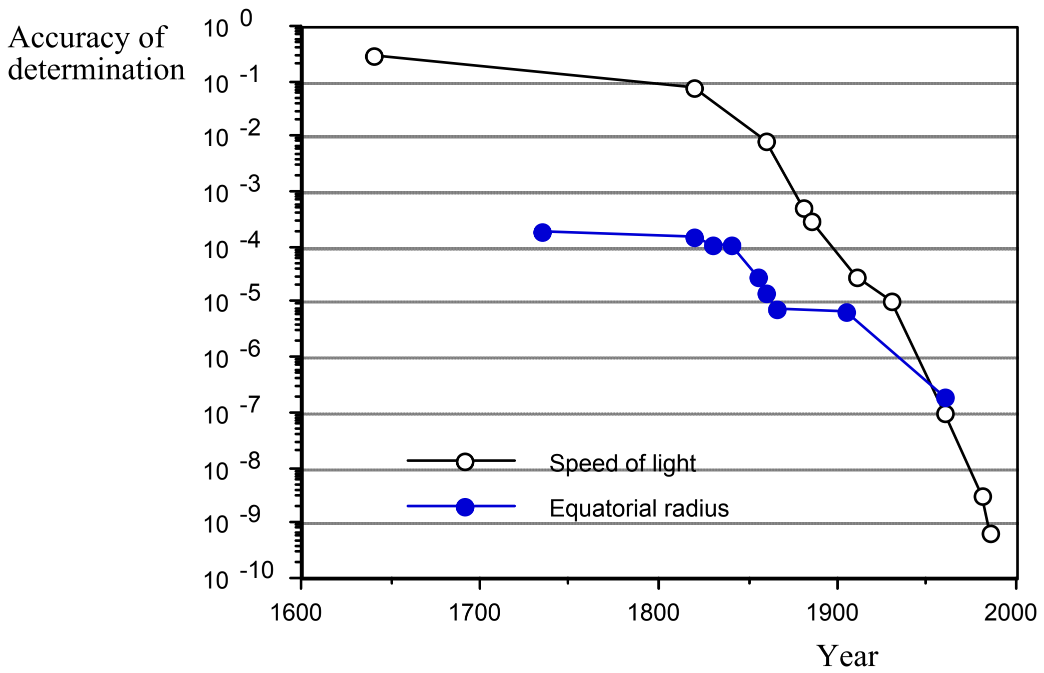

Figure 1.3 *- Trends in accuracy of determination of length and speed of light*

Future realisations of the metre may include some form of ion-trapping arrangement in which individual atoms or ions are cooled by lasers and trapped in fields, where they undergo transitions between energy levels [**13**]. It is thought that these "single oscillators" will exhibit exceptional frequency stability and purity, of the order of 10^{-18}.

Other proposals for slow-atom frequency standards include laser cooled atomic fountains [**14**] with an estimated accuracy of 10^{-16}.

It will be necessary to link the frequency of these transitions to the caesium clock, and then establish a new trapped ion standard as the frequency standard. This will allow accurate beat frequency comparison with stable laser sources, which can then serve as length standards. Thus it may require a change of viewpoint from the current notion of measuring length against wavelengths, to timing the flight of the light, using the laser as an accurate frequency standard or clock.

1.4 SECONDARY LENGTH STANDARDS

1.4.1 Modern secondary length standards

Although the unit of length is the metre, and the realisation of the metre is via the wavelength or frequency of a frequency-stabilised laser, it is inappropriate to use this primary standard for everyday measurements and so use is made of secondary length standards for less demanding measurements. There are two types of length standard in use: line standards and end standards. Examples of line standards include survey tapes, photomasks, the Prototype Metre and of course rulers. End standards include gauge blocks, length bars, the Mètre des Archives, Hoke gauges and combination bars.

The measurement of line standards usually requires an instrument which works on the same principles as, or is actually a travelling microscope which traverses the distance between two or more lines marked on the standard. Line standards range from the dimensions of micro-lithographic standards of the 1 μm size, up to 50 m survey tapes.

End standards usually take the form of bars of durable material and have flat, polished end faces, the separation between them defining the length of the standard. Calibrations of end standards are made using mechanical probes or interferometrically. End standards are widely used in industry for calibrating verniers, micrometers and for verifying the performance of Co-ordinate Measuring Machines (CMMs) [**15**]. They are more suitable for engineering measurements than line standards as they represent a “mechanical” length which can be physically probed.

1.4.2 Gauge blocks and length bars

The two most common forms of end standard in use throughout Europe are *gauge blocks* and *length bars*. These are material standards and take the form of rectangular or circular bars made to various nominal lengths according to certain standards [**16**,**17**,**18**,**19**,**20**,**21**].

Figure 1.4 *- A set of gauge blocks, 2 long series gauge blocks and 2 length bars (foreground)*

1.4.3 Gauge blocks

A gauge block is a block of rectangular section, made of durable material, with one pair of plane, mutually parallel measuring faces. The length of a gauge block at a particular point on the measuring face is the perpendicular distance between this point and a rigid plane surface of the same material and surface texture upon which the other measuring face has been wrung. ‘Wringing’ is a technique by which very flat, lapped surfaces can be made to adhere to one another by molecular attraction (see § 2.5.1).

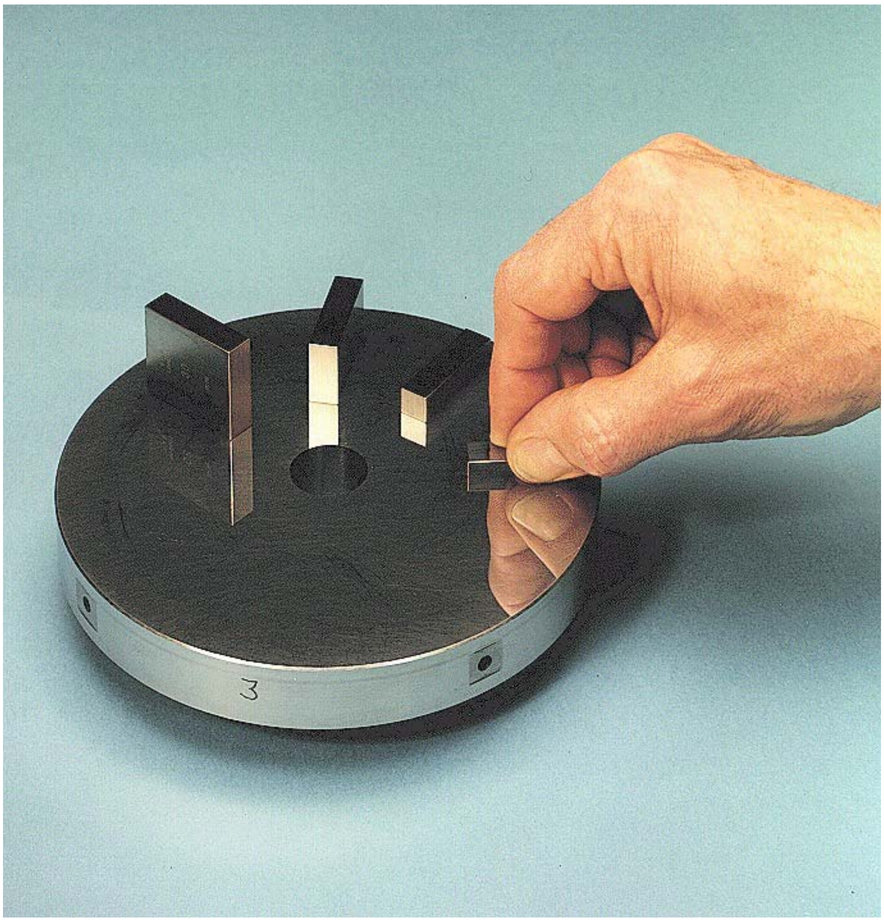

Figure 1.5 *- Gauge blocks being wrung to a platen*

The measured length of a gauge block is corrected to the reference temperature of 20 °C and standard air pressure 101 325 kPa (1 013.25 mbar). The lengths of gauge blocks up to and including 100 mm refer to the length of the gauge block in the vertical position, *i.e.* with the measuring faces horizontal. The lengths of gauge blocks over 100 mm refer to the length of the gauge block in the horizontal position, the block being supported on one of the smaller side faces without additional stress by two suitable supports, each at a distance of 0.211 times the nominal length from the ends. This is to account for prismatic compression of the gauge under its own weight, when standing vertically and 'sagging' when supported horizontally (see Appendices C & D).

1.4.4 Length Bars

Length bars are end standards of cylindrical cross-section, 22 mm in diameter, having flat, parallel end faces finished by lapping. They are made from high quality tool steel, free from non-metallic inclusions. The length of a bar is defined with the bar mounted horizontally (and referred to the standard reference temperature of 20 °C) as *the distance from the centre of one of its faces to a flat surface in wringing contact with the opposite face, measured normal to the surface*.

25 mm bars are hardened throughout their length. Bars over 25 mm up to and including 125 mm are hardened either throughout their length or at the ends only for a distance of not less than 4 mm. Longer bars are hardened at the ends only for a distance of about 6 mm and not less that 4 mm from each end.

1.4.5 Definitions and specifications for Reference Grade length bars

Additionally, the British Standard (BS 5317) places detailed specifications on length bars, depending on the grade of manufacture, as follows.

Deviation from flatness
The minimum distance between two parallel planes which just envelop the measuring face. The maximum permissible values are given in table 1.4.

Deviation from parallelism
The difference between the maximum and minimum lengths at any points on the measuring faces measured perpendicular to the surface to which one face is wrung. The maximum permissible values are given in table 1.4.

Deviation from squareness
The minimum distance between two parallel planes normal to the axis of the bar which just envelop the measuring face under consideration.

Diameter
The diameter of each bar shall be uniform within 15 µm for bars up to 300 mm in length, 25 µm for bars over 300 mm up to and including 600 mm in length, and within 50 µm for bars longer than 600 mm.

Straightness
The body shall be straight within 10 µm per 100 mm of length.

Squareness
The end faces of all grades of bars shall be square with the axis of the bar to within 1.2 µm over the diameter of the face for bars up to and including 400 mm in length and to within 2.5 µm for bars over 400 mm in length.

Nominal length	Tolerances on accuracy of faces		Tolerance on length at 20 °C
	Flatness	Parallelism	
mm	± µm	± µm	± µm
up to 25	0.08	0.08	0.08
50	0.08	0.10	0.12
75	0.10	0.16	0.15
100	0.10	0.16	0.20
125	0.10	0.20	0.25
150	0.10	0.20	0.30
175	0.15	0.20	0.30
200	0.15	0.20	0.35
225	0.15	0.20	0.40
250	0.15	0.30	0.40
275	0.15	0.30	0.45
300	0.15	0.30	0.50
400	0.15	0.30	0.65
500	0.15	0.30	0.80
600	0.15	0.30	0.95
700	0.15	0.30	1.10
800	0.15	0.30	1.25
900	0.15	0.30	1.40
1000	0.15	0.30	1.55
1200	0.15	0.30	1.85

Table 1.4 *- Tolerances on parallelism, flatness and length for reference bars according to BS 5317:1976*

1.4.6 Calibration of End Standards of length

Gauge blocks and length bars of the highest grade of accuracy are calibrated at the National Physical Laboratory (NPL). These calibrations are traceable to the definition of the metre through the use of stabilised laser wavelengths. For gauge block calibrations, this traceability is provided directly by the NPL automatic Gauge Block Interferometer [**22**].

This instrument uses calibrated frequency-stabilised lasers to relate the lengths of gauge blocks to the realisation of the metre in terms of the wavelength emitted by an iodine-stabilised laser. The instrument is computer controlled and performs corrections to take account of the variations in the refractive index of the air, and the temperature of the gauge blocks.

For length bar calibrations, use is made of the NPL Length Bar Machine. This instrument functions as a comparator in which the length of a test bar is compared to the length of a short (3 in) standard length bar which has been measured in the Gauge Block Interferometer (GBI). The instrument uses a mechanical probing system with the separation of the two probes monitored using a commercial fringe counting interferometer. Corrections are made for the variations in refractive index of the air inside the interferometer and for the thermal expansion of the length bars.

This instrument simply provides a measurement of the central length of the bar and relies on the calibrated standard for its traceability. The instrument can experience problems with alignment which can limit its accuracy.

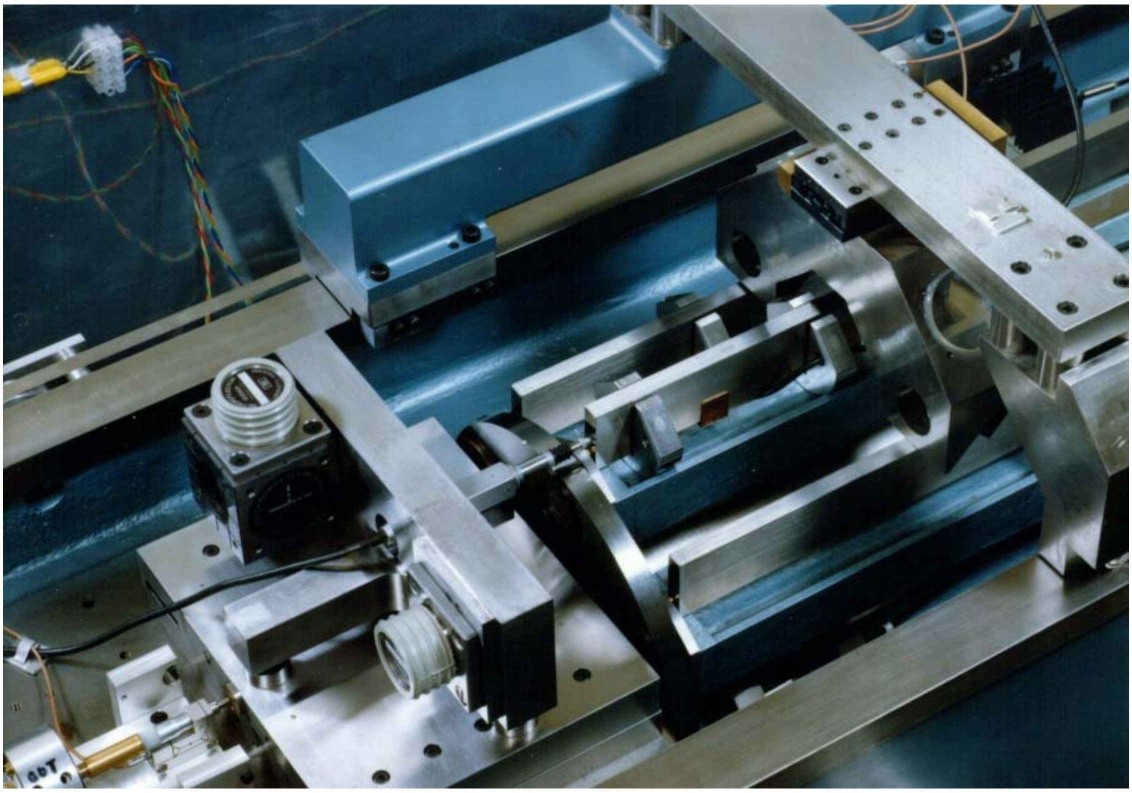

Figure 1.6 *- NPL Length Bar Machine: probes contacting a long series gauge block*

1.4.7 The rationale behind the development of the new interferometer

Emerging customer requirements showed a need for increased accuracy in length bar calibrations, especially for verifying the performance of co-ordinate measuring machines (a £20 M per annum CMM manufacturing market for the UK in 1986 [**23**]). Also there was a need to provide accurate longer standards for the NPL Length Bar Machine (over 300 mm) with measured values of flatness, parallelism and thermal expansion coefficient, and also an independent technique for verifying the performance of the LBM.

It was decided that a new instrument should be constructed to overcome as many of the limitations of the Length Bar Machine as possible and to measure additional parameters of length bars including: thermal expansion coefficient, flatness of measurement faces, parallelism of faces, as well as providing a more accurate measurement of the length of the bars. It was decided that this new interferometer should be capable of measuring both length bars and long series gauge blocks (gauge blocks over 100 mm) between 100 mm and 1500 mm in length. The interferometer could be used either as a calibration instrument, or to provide traceable standards for use in the Length Bar Machine. This new interferometer, the National Primary Length Bar Interferometer (NPLBI), is the subject of the research presented in this thesis. To put the importance of this work into context, the next section deals with the traceability chain of length measurements through the use of calibrated end standards in the UK.

1.4.8 Traceability of length bar length measurements

When length bars are measured in terms of the primary standard of length, *i.e.* the wavelength emitted by the iodine-stabilised laser, they can then be used to calibrate the lengths of other standards through comparison or can be used to verify the performance of length measuring instruments. This hierarchical system of standards represents the *traceability* of length measurements - in theory any measurement of length can be traced to the definition of the metre. By nature of the hierarchical structure and the loss of accuracy at each comparison stage, it is obvious that the higher in the pyramid, the more accurate must be the standard, with the primary standard being the most accurate (see figure 1.7).

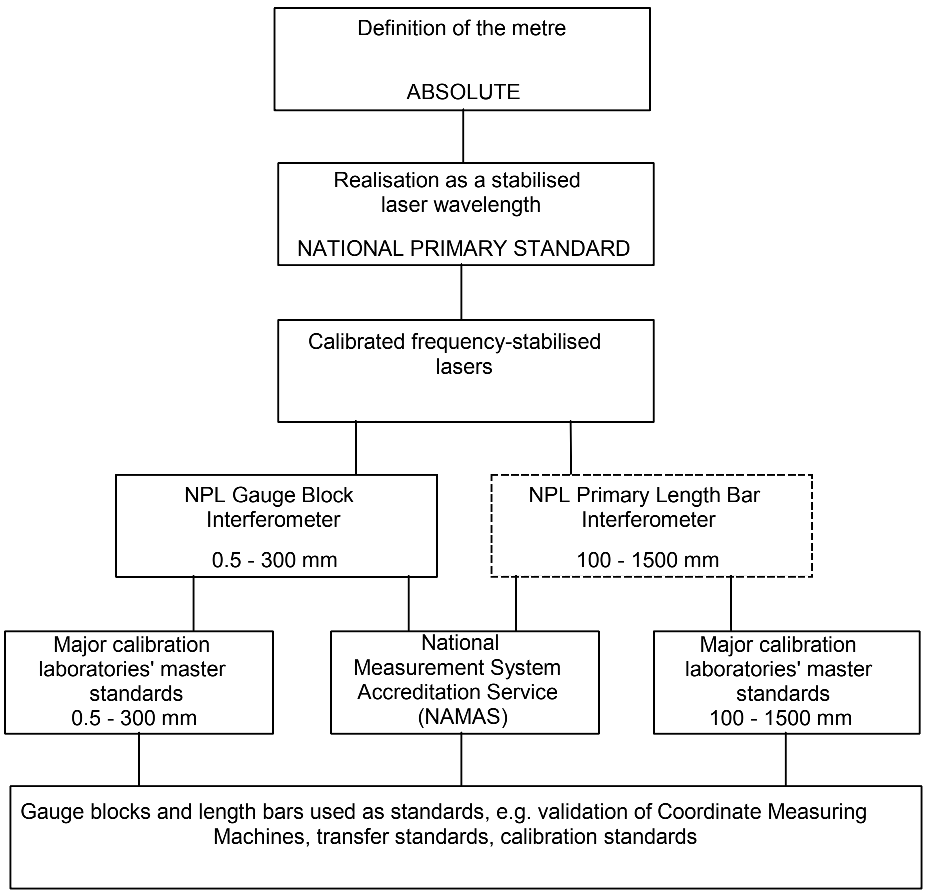

Figure 1.7 *- Traceability of length measurements*

At the head of the traceability chain is the definition of the metre. This is an absolute standard and is part of the SI system of units (see earlier). The metre is realised at NPL as a wavelength of a helium-neon laser stabilised to a saturated absorption in iodine at 632.991 398 1 nm. Details of the operation of this laser are given in § 3.2.1.3. A recent intercomparison of iodine-stabilised lasers has shown agreement at the level of 2 x 10^{-11} between lasers and has resulted in a new uncertainty being adopted for the UK realisation of the metre of ± 2.5 x 10^{-11}.

Figure 1.8 *- Iodine-stabilised He-Ne Primary laser*

As the Primary laser is the UK's Primary length standard, it follows that it is not used for routine calibration of end standards via interferometry. Instead, commercial stabilised lasers (based on an NPL design) are used after having their wavelengths calibrated against the Primary laser. These lasers are used in both the existing Gauge Block Interferometer and the National Primary Length Bar Interferometer.

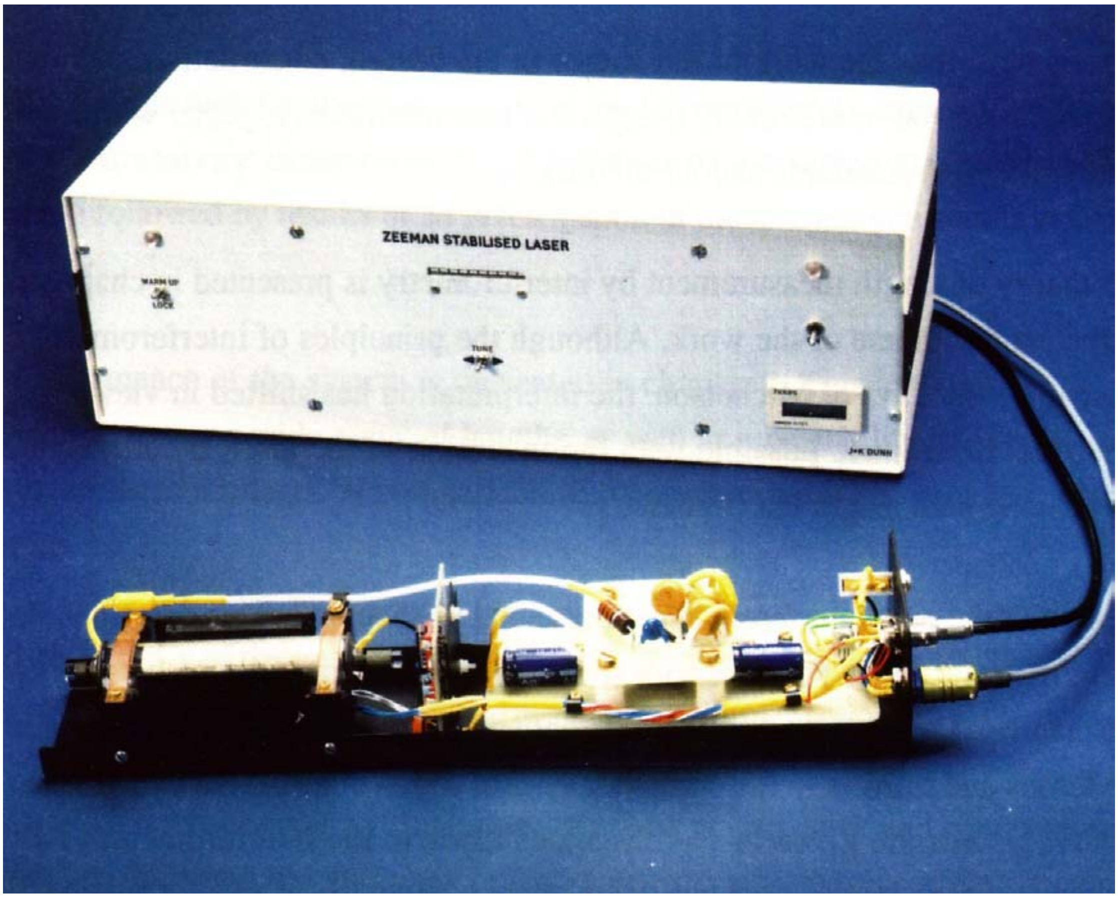

Figure 1.9 *- NPL design Zeeman stabilised laser*

Annually, NPL measures the length of some 200 length bars and 1200 gauge blocks for customers as well as providing a measurement and audit service for laboratories accredited under NAMAS (National Measurement Accreditation Service). In turn, these calibrated length standards are disseminated throughout UK industry where they are used to demonstrate traceability of length measurement for billions of pounds worth of trade.

As well as demonstrating national traceability, frequent European intercomparisons ensure that different countries' measurement systems are compatible and demonstrate traceability of length measurement to the internationally agreed definition of the metre, detailed previously in this chapter. As an example, a recent EUROMET intercomparison of gauge block measurements showed agreement between a number of European standards labs to within each lab's uncertainty budget for gauge block calibrations. For NPL this meant that the difference between gauge block lengths measured by NPL and the mean of all of the results was less than 60 nm for a 100 mm gauge block.

1.5 CONTENTS OF THE THESIS

This thesis describes the work of the author in the design, construction, commissioning and testing of the National Primary Length Bar Interferometer, together with research into the theory of length standards and interferometry.

A brief theory of length measurement by interferometry is presented in chapter 2, as this forms the central theme of the work. Although the principles of interferometry have not changed since the days of Michelson, the interpretation has shifted in viewpoint. A new approach has been made possible with the invention of the laser, an almost ideal light source for long path length interferometry.

The design of the new interferometer is presented in chapter 3. This describes the overall design of the instrument, together with some more detailed aspects such as the optics and the mechanics. The instrument itself is a collection of many components ranging from simple lenses and mirrors to the computer system which controls the entire measurement procedure. Details are given of modifications made to the interferometer to allow it to make double-ended measurements of length bars without the need for wringing.

The techniques for aligning the interferometer are dealt with in chapter 4, together with the results of preliminary checks made on the system, and various known optical defects and discrepancies.

Chapter 5 examines the analysis of interference fringes with particular emphasis on phase-stepping techniques. The techniques of fringe skeletonisation, temporal heterodyning, spatial heterodyning, Fourier Transform, phase-locking and phase-stepping (or phase-shifting) interferometry are examined. Error sources are identified and the limitations and benefits of the techniques explained. Phase-stepping interferometry is selected as the most suitable for analysis of fringes in the Primary Length Bar Interferometer. The techniques of 3, 4 and 5-position phase-stepping interferometry are examined. The 5-position technique that is selected belongs to the set of "N+1 symmetrical" algorithms. A demonstration of the errors of the technique is given, followed by a description of the implementation of the algorithm in the interferometer.

The data analysis and information processing are covered in chapter 6, including the software algorithms for phase-unwrapping and multiple-wavelength analysis. Overviews are given of the hardware and software, together with some example results.

The topic of refractive index correction of the laser wavelengths is the subject of chapter 7. This includes the operation of an air refractometer used to verify the performance of the empirical equation used to calculate the refractive index and an assessment of the stability of the refractive index within the chamber.

Chapter 8 deals with the thermal control of the interferometer chamber and the subject of thermal expansivity measurements. The design of the temperature control system is presented followed by results of an investigation of the temperature stability of the air and length bars.

The performance of the system is presented in chapter 9. This includes results of length measurement over a wide range of lengths, as well as repeatability and results of thermal expansion measurements. A comparison of measurements with other NPL equipment is reported.

An error analysis of the complete instrument is presented in chapter 10. This will form the basis of the uncertainty budget for the completed instrument.

The final chapter, chapter 11, forms an overview of the work and draws some general conclusions on length measurement by interferometry and some specific conclusions with regard to the National Primary Length Bar Interferometer.

Appendix A is a list of the optical and mechanical equipment used to construct the interferometer with details of the optical testing of the equipment in appendix B. Appendix C deals with the bending of bars supported horizontally due to their own weight and the compensation of the platen's weight. Appendix D is a calculation of the compression of length bars when standing vertically. Appendix E is a list of the electrical connections and connectors. Copies of papers published by the author are collected in appendix F.

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